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Toward High-Resolution Mechanical Spectroscopy HRMS. Logarithmic Decrement

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Abstract. In this work, we present the comparison between different methods used to compute the logarithmic decrement, δ. The parametric OMI method and interpolated DFT (IpDFT) methods are used to compute the δ from free decaying oscillations embedded in an experimental noise typical for low-frequency mechanical spectrometers. The results are reported for $\delta = 5\times10^{-4}$, f_0 = 1.12345 Hz and different sampling frequencies, f_S = 1 kHz and 4 kHz. A new YM algorithm yields the smallest dispersion in experimental points of the logarithmic decrement and the smallest relative errors among all investigated IpDFT methods. In general, however, the IpDFT methods suffer from spectral leakage and frequency resolution. Therefore it is demonstrated that the performance of different methods to compute the δ can be listed in the following order: (1) OMI, (2) YM, (3) YM_C, and (4) the Yoshida method, Y. For short free decays the order of the best performers is different: (1) OMI and (2) YM_C. It is important to emphasize that IpDFT methods (including the Yoshida method, Y) are discouraged for signals that are too short. In conclusion, the best methods to compute the logarithmic decrement are the OMI and the YM. These methods will pave the way toward high-resolution mechanical spectroscopy HRMS.

Introduction

Analysis of exponentially damped harmonic strain-response signals in mechanical spectroscopy is commonly conducted using a variety of classical methods [1-6], Discrete Fourier Transform-based methods (DFT) [3,7-11], Hilbert Transform-based methods [3,6-8,12,13], and recently a parametric method OMI (*Optimization in Multiple Intervals*) [1,2,4-6]; methods are listed in a chronological order. In a similar way as computations of the resonant frequency f_0 the integral transform-based methods (i.e. DFT, Hilbert transform) show inherent performance limitation due to frequency resolution [1,7,9-13]. That is why discussion on computations of the δ and the f_0 cannot be put forward separately (these problems are discussed in this volume in two contributions.) The δ and the f_0 are jointly computed here according to the OMI method and a number of interpolated DFT (IpDFT) methods [1,9-11].

The expectation that the OMI outperforms the classical methods was demonstrated in [1,2,4-6]. It was also shown that the OMI can successfully be used to analyze short and very short free decaying signals [1,2,5]. Moreover, the OMI method performs very well for exponentially damped harmonic signals embedded in an experimental noise and biased by an offset and/or the Zero-Point Drift (ZPD) effect as already shown in [4,6].

The DFT transforms free decaying oscillations from the time domain into the frequency domain, and has the very useful property that yields the δ and the f_0 independently of the baseline offset dc. In this work, we shall focus our attention on the comparison between various IpDFT methods and the results will be compared to results obtained with the ‘gold standard’, i.e. the OMI method [1]. The Hilbert transform methods [7,12,13] and the influence of the ZPD on computations of the δ [4,6,12,14-17] are omitted here due to space limitations. The results of computing the δ for a set of

100 free decays are analyzed here as a function of the length of a free decaying signal [2,4]. Finally, the IpDFT methods are listed as a function of its performance (i.e. the smallest relative error γ_δ, the smallest minimal $\gamma_{\delta\ \min}$ and the maximal $\gamma_{\delta\ \max}$ relative error in the estimation of the δ, that is, the smallest dispersion of experimental points in mechanical loss measurements, δ= f(T).)

Experimental Results and Discussion

For purely exponentially damped time-invariant harmonic oscillations embedded in an experimental noise $\varepsilon_w(t)$ recorded in a low-frequency resonant mechanical spectrometer the logarithmic decrement can be determined from Eq. (1) using only the digitized data $A_i(t)$ and t_i from free decaying signal:

$$A(t) = A_0 e^{-\delta f_0 t}\cos(2\pi f_0 t + \varphi) + \varepsilon_w(t) + dc\,, \tag{1}$$

where A_0 is the maximal strain amplitude, t is a continuous time in seconds, $-\pi < \varphi \le \pi$ is the phase in radians, and dc is an offset. The noise $\varepsilon_w(t)$ corresponds here to the signal-to-noise ratio S/N= 32 dB [1,2]. In this work, we shall compare the performance of different IpDFT methods and the OMI method for free decaying oscillations characterized by $\delta = 5\times10^{-4}$, $f_0 = 1.12345$ Hz, and digitized with two sampling frequencies: $f_S = 1$ kHz and $f_S = 4$ kHz. Although this work is devoted to a low damping level ($\delta = 5\times10^{-4}$) our conclusions are also valid for higher damping levels.

The purpose of this work is to show dispersion in the δ values computed according to different methods as a function of the length of analyzed signals embedded in an experimental noise. A small but frequently inevitable offset is allowed in computations. It is important to emphasize here that Yoshida *et al.* [9] were the first to suggest an interpolated DFT method to compute the logarithmic decrement for exponentially damped harmonic signals. This fact was clearly pointed out in [11] where a survey of different IpDFT methods and a discussion on systematic errors caused by spectral leakage and a picket fence effect is given. Let us recall that the Yoshida method (Y) uses four DFT bins [9] while the Agrež method (A) uses three DFT bins and the Hann window [10]. The Y_L method differs from the Y method by the use of a fixed length of the signal $A(t)$. The YM method uses four optimal values of DFT bins and a rectangular window whereas the YM_C differs from the YM method by using a complete number of oscillations.

Fig. 1 shows dispersion in computed δ values obtained according to different methods: OMI, YM, YM_C, Y_L, and Y for different lengths of free decays. The results obtained for each method are obtained for 100 sets of experimental points (A_i, t_i) (computed δ values are displayed vertically to illustrate dispersion of the results; the results obtained for different methods are shifted horizontally for clarity of presentation.)

Figs. 1 and 2 clearly illustrate that the OMI (the first data plotted vertically from the left side) outperforms IpDFT methods (2^{nd} - 5^{th} line) for all lengths of analyzed signals. Moreover, the original Yoshida method Y [9] (1^{st} data from the right) usually generates the highest dispersion in δ values and returns the highest relative error γ_δ (Figs. 1, 2; see e.g. L_{osc} = 15, 40, 60, 70, 90, 100.)

It should be pointed out that the Agrež method [10] retrieves wrong values of the δ (not shown in Figs. 1-4.) This fact is brought about by the use of the Hann window in this method. That is why the Agrež method is forbidden to be used in the field of mechanical spectroscopy.

It is obvious that computed δ values are biased for signals that are too short (Figs. 1a, 2a, 3a.) In such cases only the OMI and YM_C methods return results of high quality.

Figs. 3a and 3b show the minimal $\gamma_{\delta\ \min}$ and the maximal $\gamma_{\delta\ \max}$ relative errors of the δ. All δ values computed for 100 free decays lie between these two values along the full length of the signal $A(t)$. Fig. 3 also illustrates that the order of best methods is the following: (1) OMI, (2) YM, (3) YM_C, (4) Y_L, and finally (5) the original Yoshida method, Y. For short signals the OMI and the YM_C algorithms are recommended (Figs. 1a, 2a, 3a) while other IpDFT methods (including the

Yoshida method) are strongly discouraged. The performance of all IpDFT methods reported in this work is similar for other damping levels. In all investigated cases, among all IpDFT methods, the YM method - developed in this work - returns the smallest dispersion of computed δ values and the smallest standard deviation, σ. It is important to reiterate that the OMI outperforms the YM method. It is also noteworthy that an increase of the sampling frequency f_S from 1 kHz to 4 kHz reduces dispersion of computed δ values by around 50 % for all lengths of the free decaying oscillations. The sampling frequency f_S turns out to be an important factor which can increase the performance of the OMI and IpDFT methods (this conclusion is also valid for short signals [1].)

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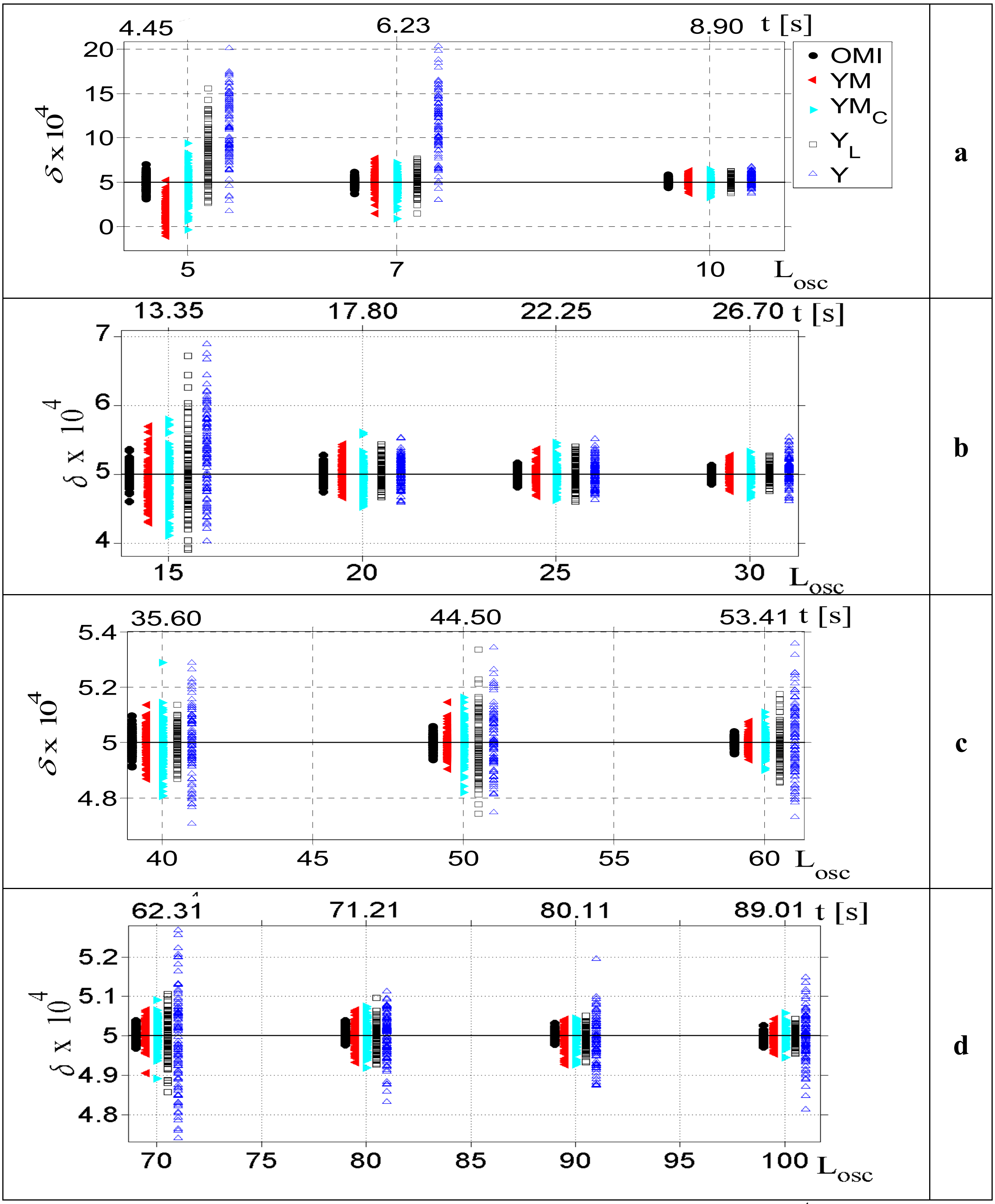


Fig. 1. Dispersion of 100 δ values for 100 free decaying oscillations ($\delta = 5\times10^{-4}$, $f_0 = 1.12345$ Hz, S/N= 32 dB, and $f_s = 1$ kHz) computed according to: OMI, YM, YM_C, Y_L, and Y methods for different lengths of the signal.

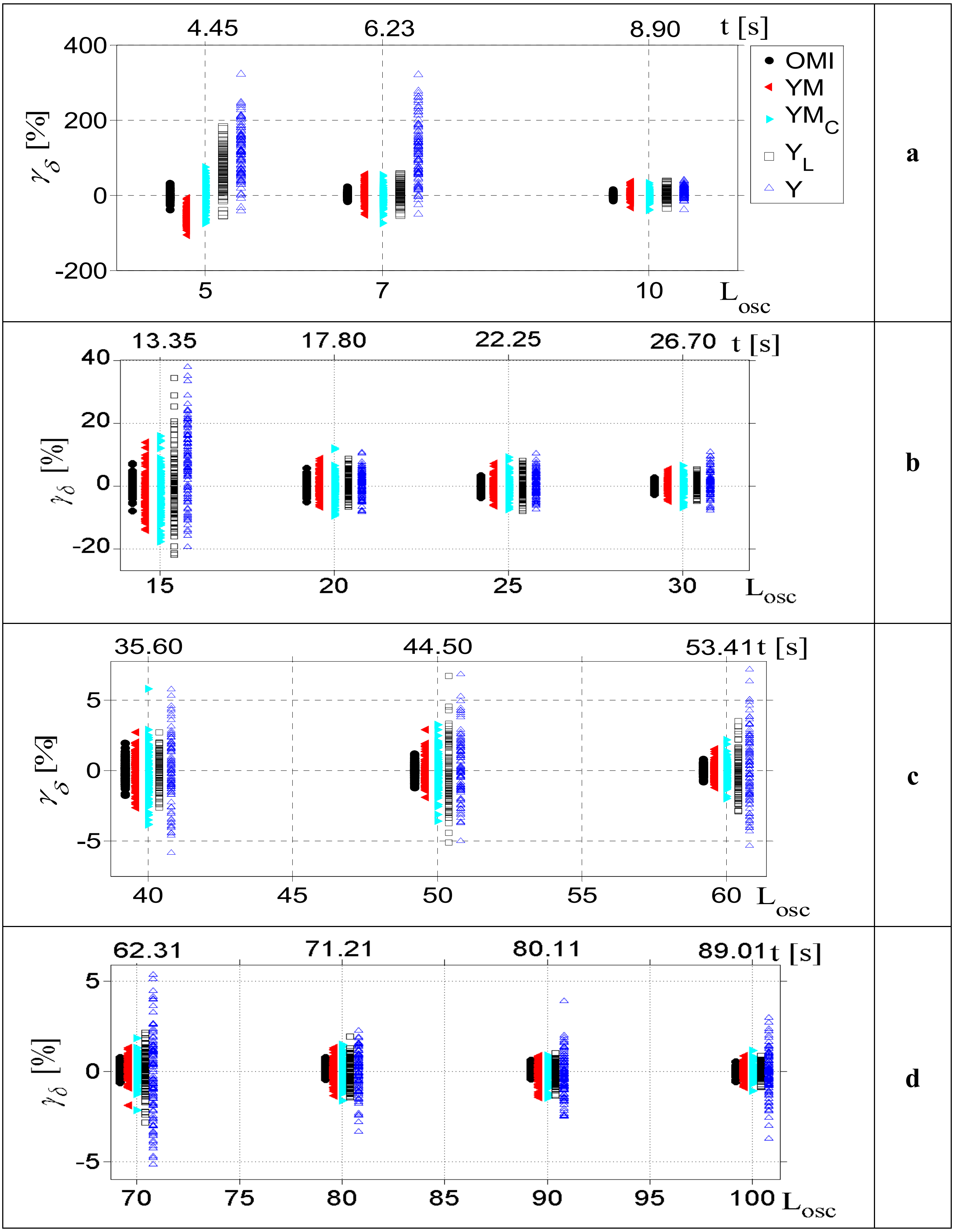
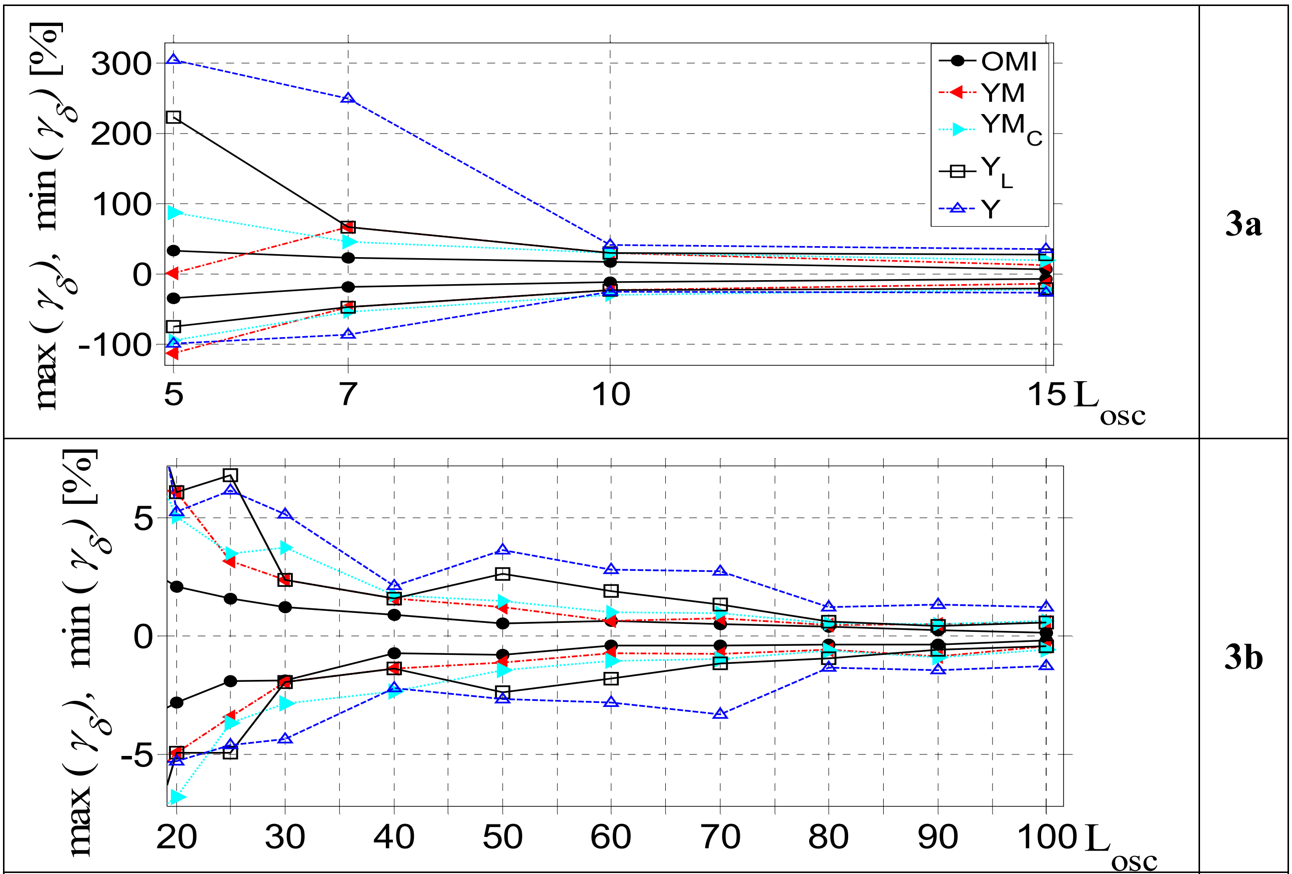


Fig. 2. Variation of the relative error for computed δ values obtained for 100 free decaying oscillations (δ= 5×10^{-4}, f_0 = 1.12345 Hz, S/N= 32 dB, f_s = 1 kHz) according to different methods: OMI, YM, YM_C, Y_L, and Y for different lengths of the signal.



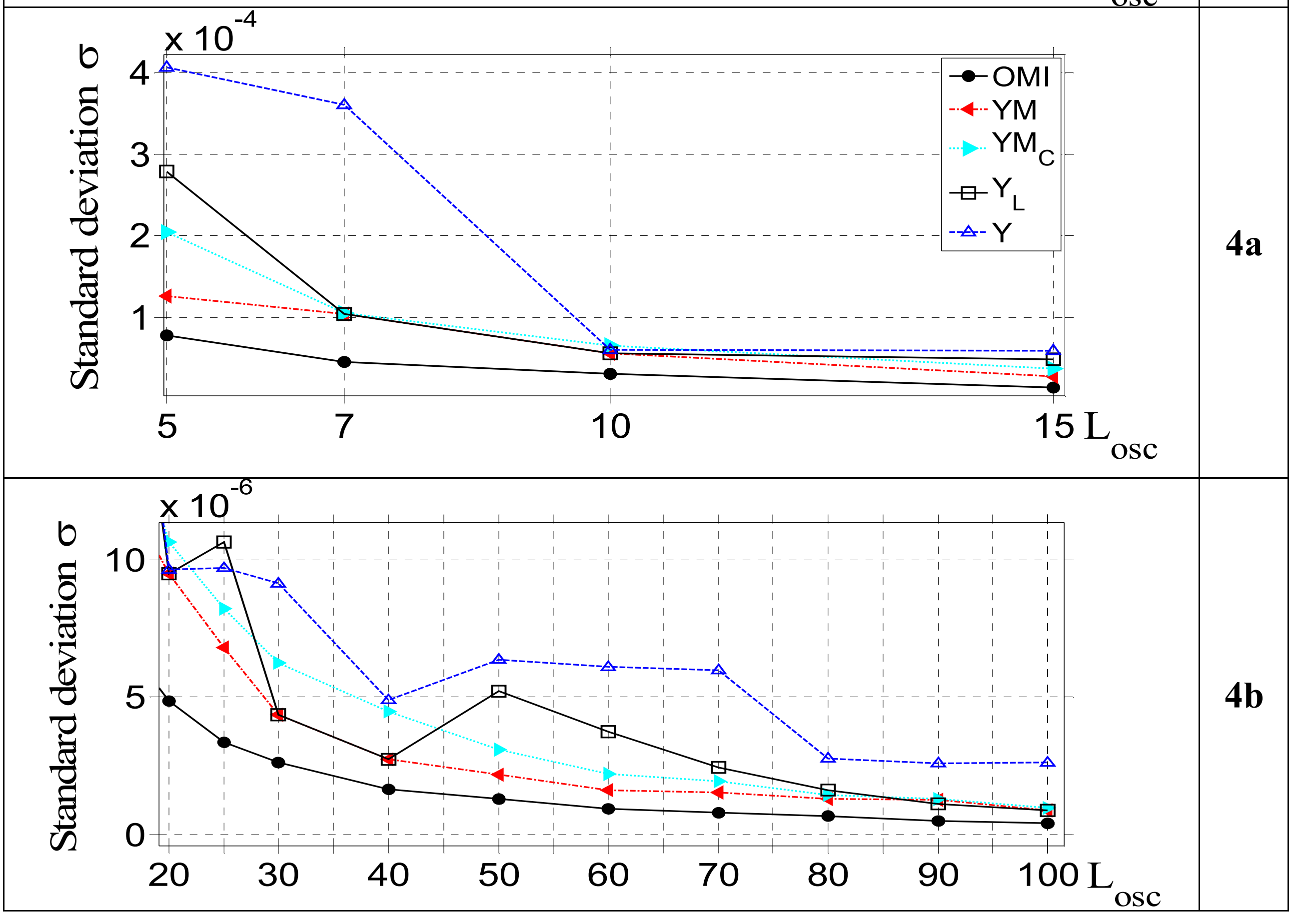


Fig. 3. The minimal $\gamma_{\delta\,\min}$ and the maximal $\gamma_{\delta\ \max}$ relative errors for computations of the logarithmic decrement δ according to different methods as a function of the length of free decaying oscillations.

Fig. 4. Variation of the standard deviation σ as a function of the length of free decaying oscillations.

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