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Recent Advances in Determination of the Logarithmic Decrement and the Resonant Frequency in Low-Frequency Mechanical Spectroscopy

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Abstract. The advantages of the OMI algorithm to compute the logarithmic decrement and the resonant frequency from free decaying oscillations is reported. The OMI algorithm is proved to be the best solution in the computation of the logarithmic decrement and the resonant frequency for high damping levels.

Introduction

In this paper we present the advantages of the OMI algorithm (Optimization in Multiple Intervals) [2 - 3] used in the computation of the logarithmic decrement δ and the resonant frequency f_o for high damping levels. A comparison between the OMI algorithm and classical methods [1 - 3] is also reported. Although the results of computations depend on several parameters such as the sampling frequency f_s of free-decaying signal, the signal-to-noise ratio S/N (a specific value for any particular mechanical spectrometer), the amplitude of oscillations A_i, the length of the decaying harmonic oscillations used for signal acquisition L (and in the computation of the logarithmic decrement), the absolute value of the logarithmic decrement δ to be measured, *a priori* defined density of experimental points, and the resonant frequency of exponentially damped harmonic oscillations f_o it is clearly demonstrated that the OMI algorithm is the best solution. In all of these instances, the OMI algorithm yields stable results, the lowest dispersion of experimental points, and the lowest relative error. The scope of this paper does not cover the case of the medium and the low level logarithmic decrement (δ below 0.01) [1, 2].

It will be also shown that the OMI algorithm yields excellent results in the computation of the resonant frequency f_o (better precision and decidedly smaller scatter in experimental points) which leads to an increase in the quality of mechanical loss spectra (both the logarithmic decrement and the resonant frequency).

The Logarithmic Decrement

The logarithmic decrement δ can be computed from several algorithms, *viz*. (1) N_osc (Number of oscillations) – from the number of N oscillations to decay from amplitude A_1 to A_{n+1} (note that in this work N is the number of oscillations for given L), (2) RA (Regression of Amplitudes) – from the height of N decaying amplitudes, (3) RS (Regression of Areas) – from the areas under a half cycle of N decaying oscillations, and (4) OMI – Optimization in Multiple Intervals. The computing algorithms used in low-frequency resonant mechanical spectroscopy are described elsewhere [1 - 4]. In the following sections the precision in the computation of the logarithmic decrement δ and the resonant frequency f_o will be compared for the algorithms mentioned before.

The OMI Algorithm

The OMI algorithm returns the logarithmic decrement δ and the resonant frequency f_o of exponentially damped pure harmonic oscillations [2, 3]. The OMI algorithm fits the following parameters A, β, f, φ, C of the theoretical function of damped harmonic oscillations $a(t)$

$$a(t) = A \cdot \exp(-\beta \cdot t) \cdot \cos(2 \cdot \pi \cdot f + \varphi) + C \tag{1}$$

to the experimental data $\{(t_i, a_i):\ i = 1 \ldots n\}$. f is the frequency of harmonic oscillations, $\beta = \delta \cdot f$, t is time, φ is the phase, and A, C are constants. t_i and a_i are time and the amplitude of the i-th sample, respectively. n denotes the total number of digital samples. The Lavenberg-Marquardt (L-M) method is usually used to minimize the nonlinear least-square function

$$S(A, \delta, f, \varphi, C) = \sum_{i=1}^{n} [\, a_i - a(t_i)\,]^2 \,. \tag{2}$$

Initial estimates of the fitting parameters $A_o, \beta_o, f_o, \varphi_o, C_o$ can be readily estimated. The initial values are returned to the vector of starting values $[A, \beta, f, \varphi, C\ ]$.

In the first step, Eq. (2) is minimized for the first cycle of damped oscillations. The fit result is returned to the vector of starting values. The second interval of experimental data contains higher number of experimental points (selection of the number of experimental points for the second interval depends on the value of the logarithmic decrement). Equation (2) is minimized for the second interval and the fit result is returned to the vector of starting values. The number of experimental data is multiplied by a parameter from the range 1.1 to 2 in the following interval, *etc.* The process is repeated until the last interval of experimental data contains all the experimental points $\{(t_i, a_i):\ i = 1 \ldots n\}$. When this occurs, the process has converged giving the final values for the parameters A, β, f, φ, C [2, 3]. It is not difficult to show, by means of the analysis of the global minimum, that the final solution is unique (the logarithmic decrement and the resonant frequency is unequivocally found). A detailed account of the OMI algorithm has been given elsewhere [2, 3].

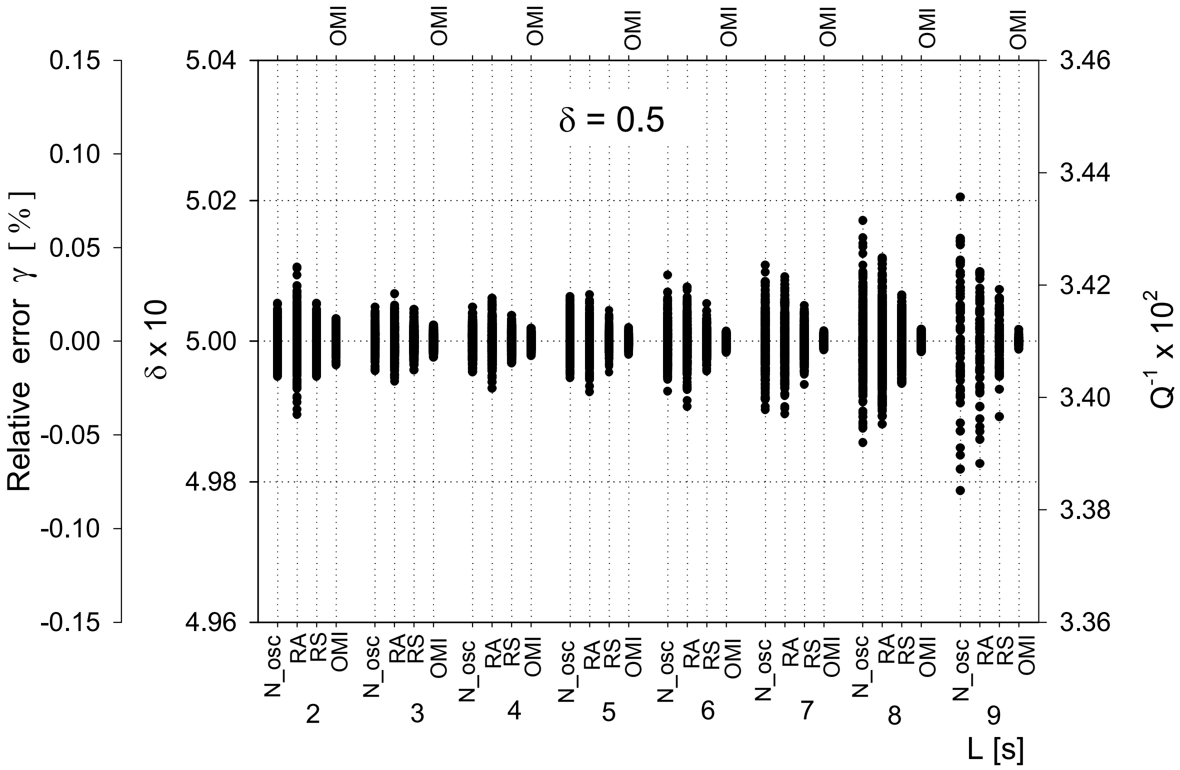
Computation of the Logarithmic Decrement

Figure 1 shows variation of the logarithmic decrement δ, the relative error γ (Fig. 1 a) and the standard deviation σ (Fig. 1 b) computed for the N_osc, RA, RS and OMI algorithms for high damping level, $\delta = 0.5$. Computations were performed for a set of 400 measurements.

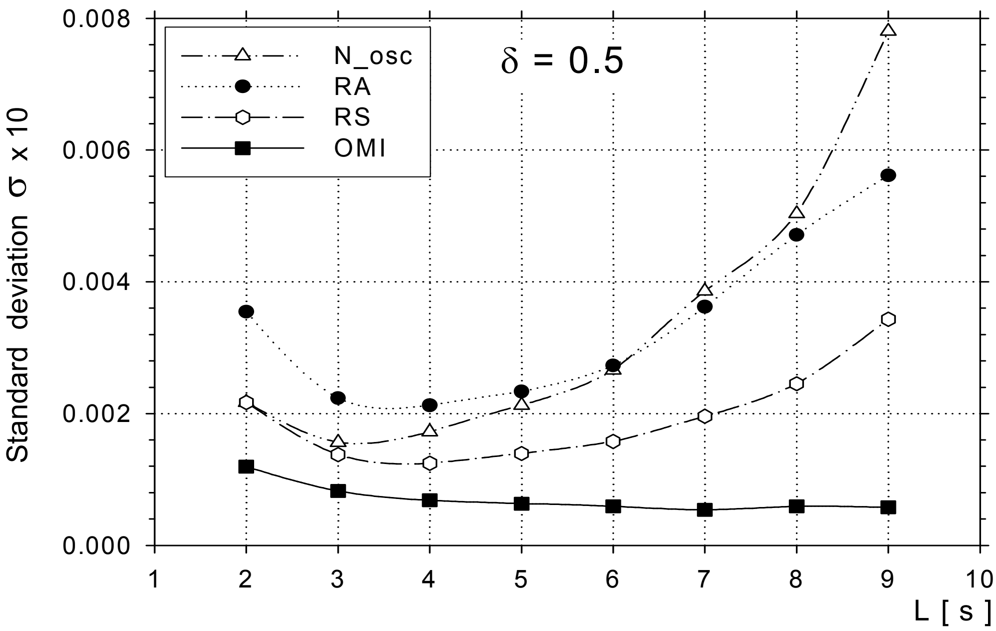
The OMI algorithm shows unequivocally that its performance is superior as compared to the classical algorithms (for long and short acquisition times). The relative error and the standard deviation depends on several parameters: (1) the length of the decaying oscillations L used for signal acquisition, (2) the sampling frequency f_s, the signal-to-noise ratio S/N, and the resolution of the A/D data acquisition board used for signal acquisition, and (3) amplitude of the decaying oscillations $a(t)$. In this work it is tacitly assumed that exponentially damped harmonic oscillations are purely symmetrical (the 'zero-point drift' ZPD is negligible [4]).

Figure 2 illustrates variation of the logarithmic decrement δ, the relative error γ (Figs. 2 a, 2 b) and the standard deviation σ (Fig. 2c) computed for the N_osc, RA, RS and OMI algorithms for high damping level, $\delta = 0.05$. Computations were performed for a set of 400 measurements. Superiority of the OMI algorithm is clearly visible for short acquisition times. This is why the OMI algorithm yields lower dispersion in experimental points and higher density of experimental points. Figure 2 also illustrates how to find the best acquisition time L (see Figs. 2 a, 2 c). The point we should like to emphasize is that excellent computing result can be obtained from the OMI algorithm for a few oscillations (3 – 6 oscillations). It turns out that further increase in the acquisition time L

(see Figs. 2 b, 2 c) does not yield substantial improvement in increasing computing precision and decreasing dispersion of experimental points. The performance of the OMI algorithm was tested for all acquisition and experimental parameters described in [2].



a)



b)

Fig. 1. (a) Variation of the logarithmic decrement δ, the relative error γ, and (b) the standard deviation σ computed according to N_osc, RA, RS and the OMI algorithm as a function of the acquisition time L. $\delta = 0.5$, sampling frequency $f_s = 5$ kHz, $S/N = 38$ dB.

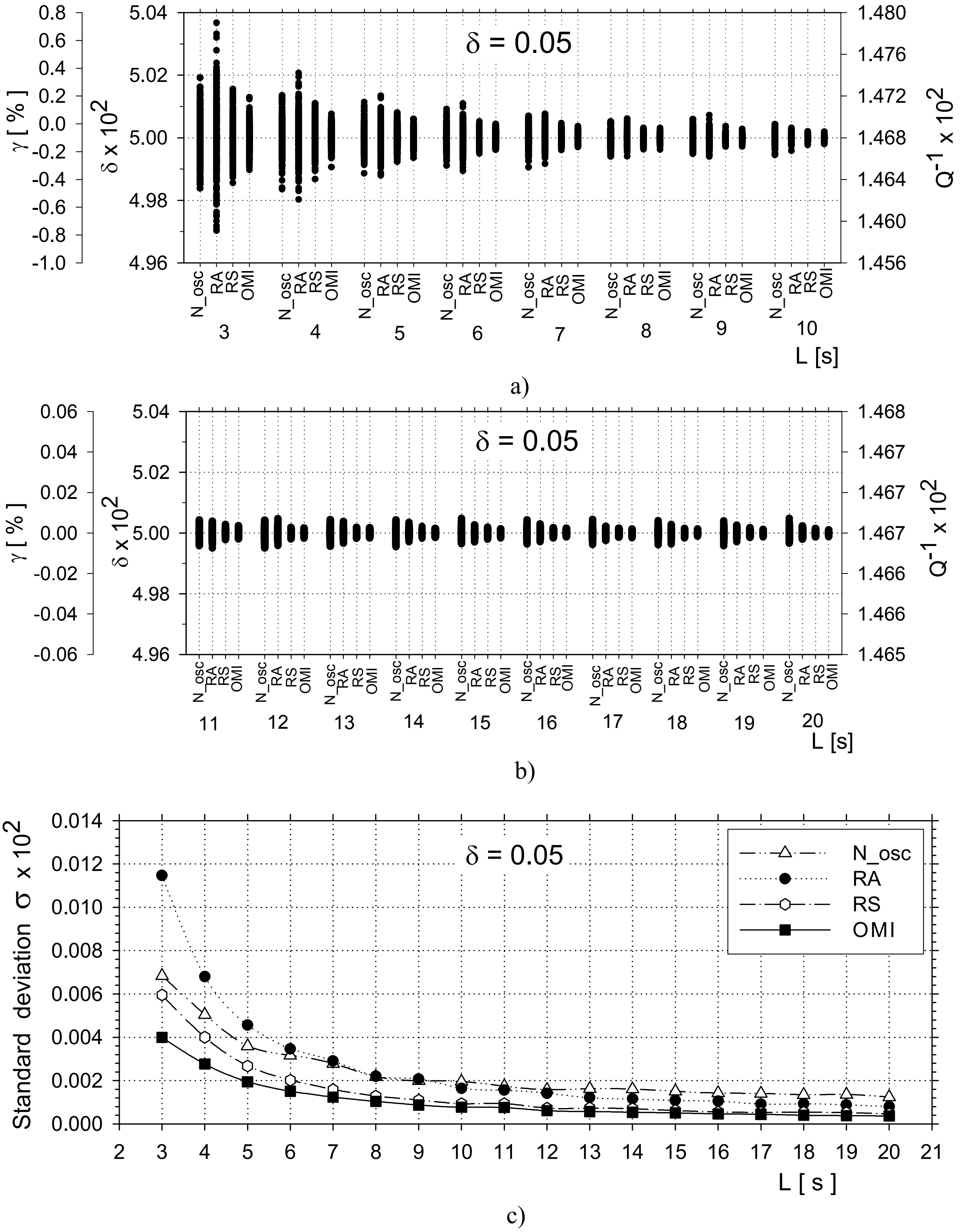


Fig. 2. (a), (b) Variation of the logarithmic decrement δ, the relative error γ, and (c) the standard deviation σ computed according to N_osc, RA, RS and the OMI algorithm as a function of the acquisition time L. δ = 0.05, sampling frequency f_s = 5 kHz, $S/N = 38\,\text{dB}$, resonant frequency $f_o = 1$ Hz.

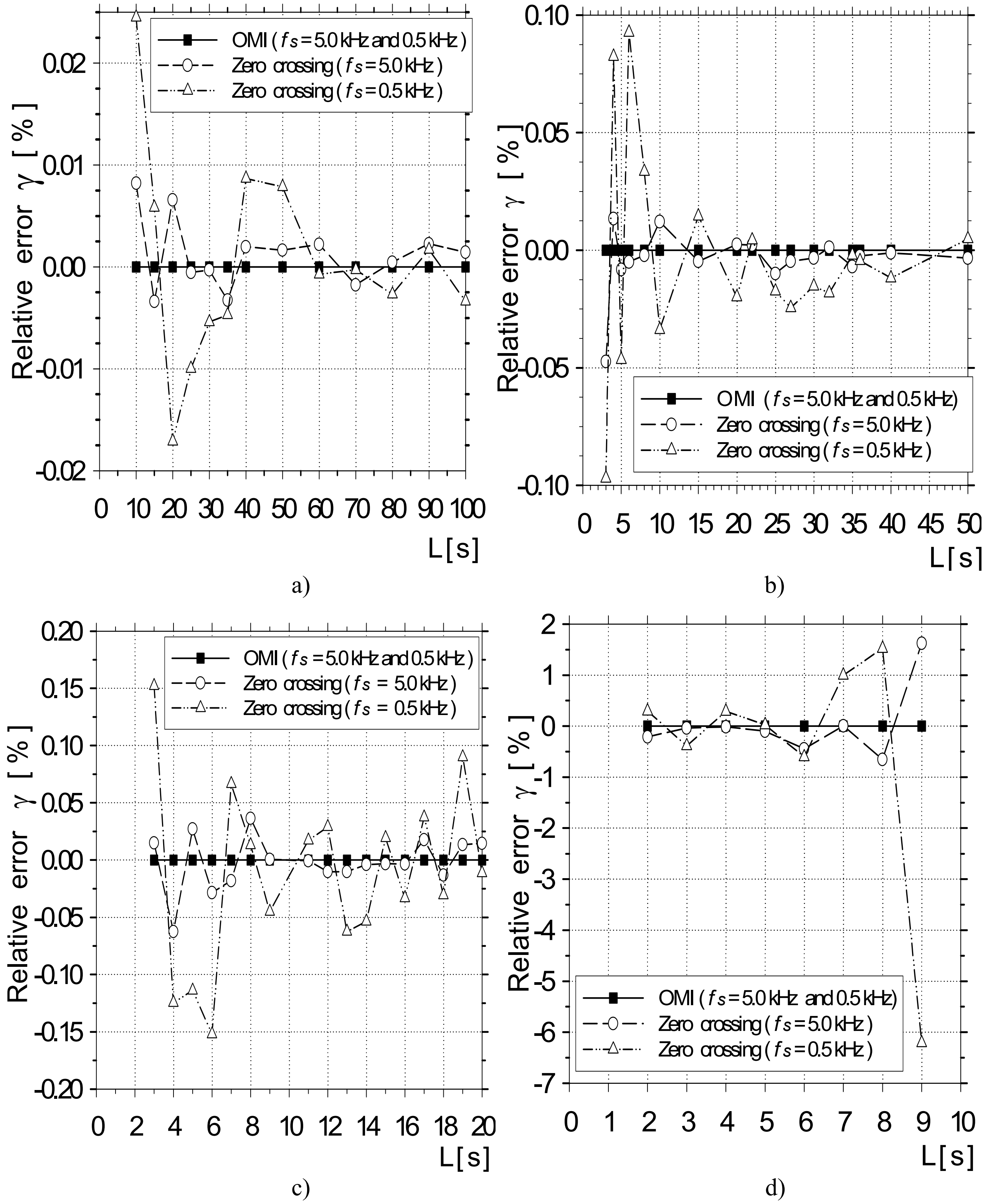


Fig. 3. Relative error γ in the computation of the resonant frequency according to the OMI algorithm (□) and the 'zero crossing' method for two sampling frequencies: f_s = 5 kHz (○) and f_s = 0.5 kHz (Δ). (a) δ = 0.0005, (b) δ = 0.005, (c) δ = 0.05, (d) δ = 0.5. Calculations were performed for a set of 400 measurements; $S/N = 38$ dB, resonant frequency f_o = 1 Hz.

Computation of the Resonant Frequency

Figure 3 illustrates precision in the calculations of the resonant frequency (f_o from 0.5 Hz to 5 Hz) as a function of the acquisition time L obtained from the OMI algorithm and the 'zero crossing' method (optimized in this work) for different levels of the logarithmic decrement. The OMI algorithm yields: (1) the best estimation of the resonant frequency, (2) the lowest relative error in the estimation of the resonant frequency, (3) decidedly better results for short, average, and long acquisition times, (4) stable computing results. It is worthwhile to emphasize that the superiority of the OMI algorithm turns out to be independent of the sampling frequency (f_s from 0.1 kHz to 5 kHz). This is why the OMI algorithm can also be recommended for 'old' mechanical spectrometers working with low sampling frequency and/or 'old' A/D data acquisition boards. Although the computing parameters [2] used in this work were tailored to obtain the smallest available error the differences between the relative errors in the calculations of the resonant frequency from the OMI and the 'zero crossing' can be 1 - 3 orders higher for a particular mechanical spectrometer. Such differences are usually induced by the acquisition and the experimental parameters discussed in [2]. In all investigated cases the OMI algorithm yields the best results.

Let us recall [2, 3] that knowing theoretical relationship between known computation error for a chosen sampling frequency in a mechanical spectrometer one can readily predict the computation error for other sampling frequency (see Fig. 3 in [2]) and the signal-to-noise ratio S/N [3]. This is why one can easily explain 'small' differences shown in Fig. 3. The effect of the sampling frequency and the signal-to-noise ratio is discussed in [1 - 3]. It is interesting to note that for the low and the medium damping levels the acquisition time cannot be reduced in classical algorithms. Wrong selection of the acquisition time L (or amplitudes A_1 and A_{n+1}) leads to an increase in the computation error of f_o (Fig. 3) and δ (Figs. 1, 2). For high damping levels classical algorithms generate high dispersion in experimental points for too long acquisition time (see Fig. 3 d). This aspect of mechanical spectroscopy has received scant attention to date and deserves more. It can be concluded that the OMI algorithm can be successfully used to make high-precision measurements of the logarithmic decrement and the resonant frequency in high-damping materials (HDM).

Conclusions

The optimal strategy in the computations of the logarithmic decrement δ and the resonant frequency f_o for the high damping level is reduced to the selection of the computing algorithm. It is concluded that for the high damping level the OMI algorithm always yields the lowest relative error γ, and the lowest standard deviation σ in the computations of the δ and the f_o. It also yields the smallest dispersion of experimental points as compared to the classical methods. The OMI algorithm provides better detection of fine variations in the logarithmic decrement and the resonant frequency and allows fast and precise detection of huge variations in the resonant frequency and the logarithmic decrement observed during phase transformations and other mechanical loss phenomena observed in viscoelastic, viscoplastic and anelastic materials.

Acknowledgments

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